

THE ORIGIN OF ULTRA-COMPACT BINARIES

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ABSTRACT

We consider the origin of ultra-compact binaries composed of a neutron star and a low-mass (about $0.06 M_{\odot}$) white dwarf. Taking account of the systemic losses of mass and angular momentum, we find that a serious difficulty exists in the scenarios which involve tidal captures of a normal star (a main sequence star or a red giant) by a neutron star. This difficulty can be avoided if a red giant star is captured by a massive white dwarf ($M \gtrsim 1.2 M_{\odot}$), which becomes a neutron star through the accretion induced collapse.

Subject headings: stars: binaries — stars: X-ray — stars: neutron — stars: white dwarfs

I. INTRODUCTION

Recently, Stella, Friedhorsky, and White (1987) discovered a 685 s orbital period of the X-ray binary 4U1820-30 in the globular cluster NGC6624. They suggested that the binary system consists of a neutron star and a low-mass ($0.055 M_{\odot}$) helium white dwarf. If it is true, 4U1820-30 is the first neutron star-white dwarf system has been ever discovered. Verbunt (1987) and Bailyn and Grindlay (1987) discussed the origin of such systems, which involve tidal capture of a normal star (main sequence star or red giant) by a neutron star.

Verbunt (1987) studied probabilities of tidal capture by neutron stars in globular clusters and concluded that 4U1820-30 may be a result of the collision between a neutron star and a red giant. If a red giant is captured in the globular cluster, the mass of the resultant helium white dwarf should be larger than $\sim 0.2 M_{\odot}$, because the helium core in the red giant becomes the white dwarf. According to his conclusion, the present 4U1820-30 should be a result of mass transfer from the white dwarf to the neutron star driven by the gravitational wave radiation. However, Hachisu, Eriguchi, and Nomoto (1987) have shown that if the mass of the white dwarf in a neutron star-white dwarf binary is larger than about $0.1 M_{\odot}$, the white dwarf is disrupted in a time scale of 10^4 yr after it fills its inner critical Roche lobe. A similar conclusion was suggested by Bailyn and Grindlay (1987). In order to avoid this difficulty, Bailyn and Grindlay (1987) proposed the scenario that starts from a tidal capture of a main-sequence star by a neutron star.

In this paper, we examine the origin of 4U1820-30 by taking quantitatively into account the systemic angular momentum loss and the physical condition in the contact phase of binaries. We will show that after tidal capture of a normal star (secondary) by a compact object (primary) and subsequent loss of the hydrogen envelope of the normal star, the remaining helium core mass is larger than $0.18 M_{\odot}$ for a reasonable mass range of the primary. Therefore, if the primary is a neutron star, the secondary will be disrupted. On the other hand, if the primary is a white dwarf and the helium core mass of the secondary is less than $0.25-0.30 M_{\odot}$ (depending on the mass of the primary), the binary system can survive. The difference of mass limit between neutron star and white dwarf cases arises from the fact that the critical mass accretion rate which makes an extended envelope around the primary is higher for a white dwarf than for a neutron star. Further estimation of mass transfer rate to induce a collapse of the primary white dwarf allows us to find a possible progenitor system.

Based on the above results, we conclude that the ultra-compact neutron star-white dwarf X-ray binary 4U1820-30 would be evolved from a binary system originally formed by tidal capture of a red giant by a massive white dwarf. After the hydrogen envelope of the red giant is removed, this binary system becomes a double white dwarf system. When a certain amount of mass is accreted onto the massive white dwarf due to the mass transfer driven by the gravitational wave radiation, the massive white dwarf will collapse to a neutron star triggered by electron

capture on ^{24}Mg and ^{20}Ne (Miyaji et al. 1980; Miyaji and Nomoto 1987) or by carbon ignition in a solid core (Isern et al. 1983). If the mass of the less massive white dwarf is less than about $0.1 M_{\odot}$ at this birth of the neutron star, the system can evolve to a system like 4U1820-30 without a disruption of the secondary. Such accretion induced collapse of a massive white dwarf was also suggested for the formation mechanism of other low-mass X-ray binaries by Taam and van den Heuvel (1986).

II. FORMATION OF HELIUM WHITE DWARFS BY TIDAL CAPTURE

In this section, we consider the evolution of a normal star (main sequence star or red giant) tidally captured by a compact object (neutron star or white dwarf) until the hydrogen rich envelope is stripped.

The probability of tidal capture by a compact object in globular cluster cores has been estimated by Verbunt (1987). The tidal capture takes place when the periastron of the orbit is less than three times the radius R_2 of the captured secondary (Fabian, Pringle, and Rees 1975; see also Verbunt 1987). In this paper, therefore, we assume the initial separation of the binary a_0 is less than $6 R_2$.

Just after the capture, there are following three cases depending on whether the radius of the captured star is larger than the radii of its inner and outer critical Roche lobes R_2^* and R_2^{**} , respectively.

I) If $R_2 > R_2^{**}$, an unstable mass overflow from the outer critical Roche lobe takes place as illustrated in Figure 1.

II) If $R_2^* < R_2 < R_2^{**}$, the mass transfer from the secondary to the primary takes place but the systemic loss from the outer critical Roche lobe is not initiated.

III) If $R_2 < R_2^*$, the mass transfer does not take place until the secondary evolves (or the system shrinks by the gravitational wave radiation) to fill its inner critical Roche lobe.

The effective radii of the inner and the outer critical Roche lobes are approximated by the following expressions (Eggleton 1983; Hachisu, Saio, and Nomoto 1987):

$$R_2^* = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})} a, \quad (1)$$

$$R_2^{**} = f(q) R_2^*, \quad (2)$$

$$f(q) = \begin{cases} 0.963 + 0.266(1/q^2 + 0.766/q + 0.01924)^{1/2} & \text{for } q > 1, \\ 1 + 0.314[1 - (1-q)^{1.73}]^{0.27} & \text{for } q \leq 1, \end{cases} \quad (3)$$

where a is the separation of the binary and $f(q)$ is a function of the mass ratio $q = M_2/M_1$, where M_1 is the mass of the primary and M_2 is the mass of the secondary, respectively. We will consider the above three cases separately in the following subsections.

a) Case I

If $R_2 > R_2^{**}$, the mass outflow from the outer critical Roche lobe begins so that it carries away the orbital angular momentum from the system. As a result, the separation and hence the outer critical Roche lobe decrease, which enhances the systemic mass loss further as shown below. This dynamical process will be stopped when the most of the hydrogen rich envelope of the secondary is lost and its radius is less than the radius of the outer critical Roche lobe. If the core mass of the secondary is too small, however, the radius of the helium core exceeds the radius of the outer critical Roche lobe and hence it will be

disrupted completely. In other words, there is a minimum core mass for the dynamical systemic mass loss to be stopped.

The relation between the systemic losses of the angular momentum and mass may be expressed as

$$\frac{\dot{J}}{\dot{M}} = \ell a^2 \Omega = \ell \left(\frac{1}{M_1} + \frac{1}{M_2} \right) J, \quad (4)$$

where J is the orbital angular momentum, M is the total mass, i.e., $M=M_1+M_2$, Ω is the orbital angular velocity, and ℓ is a numerical factor of order unity. This value of ℓ was estimated to be 1.7 by hydrodynamical simulations of the mass loss (Sawada, Hachisu, and Matsuda 1984). If the outflow velocity is smaller than the orbital velocity of the system, the gas goes out through around the outer Lagrangian points L_2 and L_3 . Then, the value of ℓ hardly depends on the various physical conditions of the binary and it lies between 1.6 and 1.7. (This value of ℓ was also estimated to be 1.7 by Nariai [1975] and to be 1.8-1.95 by Flannery and Ulrich [1977] based on the restricted three-body particle calculations.) In this paper, we adopt $\ell=1.7$. It should be noted here that the concept of the (outer critical) Roche lobe, which is based on the zero-velocity assumption, is still valid as long as the outward velocity is small compared with the orbital velocity.

Although the systemic mass loss proceeds in a dynamical time scale, the mass accretion rate onto the primary is limited by the critical rate which corresponds to the Eddington luminosity ($\sim 10^{-8} M_\odot \text{ yr}^{-1}$ for a neutron star and $\sim 10^{-5} M_\odot \text{ yr}^{-1}$ for a white

dwarf), because the accreted matter forms a hot envelope around the primary and eventually it becomes a common envelope (Hachisu, Eriguchi, and Nomoto 1986a). Therefore, we can neglect the mass transfer between the components and assume $\dot{M}=\dot{M}_2$ in equation (4). Integrating equation (4) and using the relation of Kepler motion

$$a = \frac{M_1 + M_2}{GM_1^2 M_2^2} J^2, \quad (5)$$

we obtain (see Nariai and Sugimoto 1976)

$$a = a_0 \left(\frac{M_{2,0}}{M_2} \right)^{2-2\ell} \frac{M_1 + M_2}{M_1 + M_{2,0}} \exp \left[-\frac{2\ell (M_2 - M_{2,0})}{M_1} \right], \quad (6)$$

where subscript 0 indicates the initial value.

The maximum value of the initial separation for case 1 is obtained by setting $R_2 = R_2^{**}$ in equation (2). In the following analysis, we assume that the initial mass of the secondary is $0.8 M_\odot$ (the turn-off mass of NGC6624). Then, assuming an appropriate value of M_1 , we obtain the maximum radii of the outer and the inner critical Roche lobes when all the hydrogen envelope is stripped off (quantities at this stage are denoted with subscript s). Figure 2 shows such maximum values of R_2^{**} and R_2^* as functions of the remaining helium core mass $M_{2,s}$. Also shown is the relation between core mass and radius for $0.8 M_\odot$ star ($X=0.7$, $Z=0.001$) obtained by Rood (1972). If the radius of the helium core, $R_{2,s}$, is larger than the outer critical Roche lobe, i.e., $R_{2,s} > R_2^{**}$, the unstable mass outflow continues and the secondary will be disrupted. When $R_{2,s}^* < R_{2,s} < R_2^{**}$, a stable

mass inflow from the secondary to the primary begins. When $R_{2,s} < R_{2,s}^*$, the secondary will reside inside the inner critical Roche lobe and the system stays detached until the gravitational wave radiation reduces the separation enough for the secondary to fill its inner critical Roche lobe again. The duration of this detached phase is estimated as (Landau and Lifshitz 1975)

$$\tau_{GR} = 1.5 \times 10^8 \left(\frac{a}{R_\odot} \right)^4 / \left(\frac{M_1}{M_\odot} \right)^3 q(1+q) \quad \text{yr.} \quad (7)$$

Then the maximum duration of detached phase is obtained from the maximum value of a_s (which is calculated from the maximum value of $R_{2,s}^{**}$ through eqs.[1] and [2]) as plotted in Figure 2. Roughly speaking, this period can always be shorter than the cosmic age when $M_{2,s} \lesssim 0.31 M_\odot$ for $M_1 = 1.4 M_\odot$.

From Figure 2, we obtain a minimum core mass (for an assumed primary mass) for which the dynamical systemic mass outflow stops when all the hydrogen envelope is stripped, i.e., $R_{2,s} < R_{2,s}^{**}$. These minimum values are $0.20 M_\odot$ for $M_1 = 1.0 M_\odot$ and $0.18 M_\odot$ for $M_1 = 1.4 M_\odot$. Because the core mass of a $0.8 M_\odot$ star around the turn-off is $\sim 0.10 M_\odot$, the secondary star must be a red giant to survive the dynamical mass loss which occurs after case I tidal capture.

b) Case II

If $R_2^* < R_2 < R_2^{**}$, the mass of the secondary overflows from its inner critical Roche lobe and accretes onto the primary. A

dynamical response of a star with a thick outer convective layer to mass loss leads to the increase in its radius. Therefore, if the radius of the inner critical Roche lobe is decreased by mass transfer from the normal star to the compact star, the mass transfer is dynamically unstable. Differentiating equation (5) under the condition of total mass conservation, we obtain

$$\frac{\dot{a}}{a} = -2(1-q)\frac{\dot{M}_2}{M_2} + \frac{2}{J}\left(\frac{dJ}{dt}\right)_{GR}, \quad (8)$$

where $(dJ/dt)_{GR}$ stands for the loss of the orbital angular momentum due to the gravitational wave radiation. Utilizing equation (1) in equation (8), we obtain

$$\frac{\dot{R}_2^*}{R_2^*} = \frac{\dot{M}_2}{M_2} H(q) + \frac{2}{J}\left(\frac{dJ}{dt}\right)_{GR}, \quad (9)$$

where

$$H(q) = \left[\frac{2}{3} - \frac{0.4q^{2/3} + q^{1/3}/3(1+q^{1/3})}{0.6q^{2/3} + \ln(1+q^{1/3})} \right] (1+q) - 2(1-q). \quad (10)$$

Since the time scale of τ_{GR} is much longer than the dynamical time scale, we can neglect this term in a discussion of dynamical stability.

The mass transfer is dynamically unstable when the value of $H(q)$ is positive, i.e., $q > 0.79$. For this case, the mass transfer proceeds in a dynamical time scale and hence creates a hot envelope around the primary and finally forms a common envelope (Hachisu, Eriguchi, and Nomoto 1986a, 1987). The mass

transfer rate during the common envelope phase is controlled by the critical accretion rate of the compact object, which is much smaller than the dynamical mass transfer rate but whose time scale is shorter than τ_{GR} . Then the common envelope phase continues until the mass ratio reaches $q=0.79$.

When $q < 0.79$, the mass transfer becomes stable after a certain amount of mass is transferred and R_2 becomes equal to R_2^* . Then the mass transfer rate is determined by either the gravitational wave radiation or the nuclear evolution of the secondary. The effect of the gravitational wave radiation may be dominant if the secondary mass is sufficiently low, for which evolutionary time scale is longer than the age of the globular cluster. Such system becomes a cataclysmic variable if the primary is a white dwarf, or an X-ray source if the primary is a neutron star (cf. Hut and Verbunt 1983). Since the X-ray sources thus formed have orbital periods much longer than that of 4U1820-30, we do not discuss such systems any further in this paper.

The effect of the nuclear evolution of the secondary is dominant when the initial mass of the secondary is $\sim 0.8 M_\odot$. For this case, by setting $\dot{R}_2^*/R_2^* = (\dot{R}_2/R_2)_{EV}$ (the evolutionary change in radius) and neglecting $(dJ/dt)_{GR}$ in equation (9) we have

$$\frac{\dot{M}_2}{M_2} = \left(\frac{\dot{R}_2}{R_2} \right)_{EV} / H(q) \quad \text{for } q < 0.79, \quad (11)$$

and the rate of the radius expansion is estimated to be $\sim 5 \times 10^{-10} \text{ yr}^{-1}$ after the turn-off and before the red giant stage. This gives the mass transfer rates of $\sim 1 \times 10^{-9} M_\odot \text{ yr}^{-1}$ for the typical

model $M_1 = 1.4 M_\odot$ and $M_2 = 0.8 M_\odot$. Since this mass transfer is driven by the growth of the helium core, the helium core grows to more than $0.2 M_\odot$ before the hydrogen-rich envelope disappears.

c) Case III

If the captured star resides inside the inner critical Roche lobe, the mass transfer does not take place until the secondary fills its inner critical Roche lobe. Once it fills its inner critical Roche lobe by its nuclear evolution or the gravitational wave radiation, the physical process for this case becomes identical to that for Case II.

Before we go to the next section, we comment on the unstable mass transfer by disk formation discussed by Bailyn and Grindlay (1987). If the accretion disk is formed at the beginning of the mass transfer, the angular momentum can be stored in the disk for a while. As a result, the increase of the separation is prevented to some extent. This effect reduces the critical mass ratio for the unstable mass transfer from $q=0.79$ down to $q=0.67$ (Bailyn and Grindlay 1987). Based on this effect, Bailyn and Grindlay (1987) proposed following scenario: If the initial mass ratio is larger than 0.67, the unstable mass transfer occurs and the rapid mass accretion onto the primary forms a common envelope. In this common envelope, a neutron star can spiral in and it finally strips off the envelope of the secondary. However, it is not likely for this mechanism to work, because after a small amount of mass ($\lesssim 10^{-10} M_\odot$ for a neutron star) is transferred in a

dynamical time scale, a hot envelope develops and engulfs the accretion disk and then the effect of accretion disk vanishes before a common envelope is formed.

III. SURVIVAL OF THE SECONDARY FROM THE COMMON ENVELOPE PHASE

a) Mass Transfer Rate

In the previous section we discussed the tidal capture formation of a close binary which consists of a neutron star or white dwarf primary and a helium white dwarf secondary. Such a binary is, in most cases, a detached system at its birth time. When the gravitational wave radiation carries away enough orbital angular momentum, the secondary fills the inner critical Roche lobe and the mass transfer from the secondary to the primary begins.

The mass transfer tends to increase the separation of the binary, while the angular momentum loss due to the gravitational wave radiation tends to decrease the separation. Therefore, a stable mass transfer may be realized if these two effects are cancelled out each other and, at the same time, the radius of the secondary remains equal to the radius of the inner critical Roche lobe. The condition can be expressed by equation (9) if we replace \dot{R}_2^* with \dot{R}_2 . Here R_2 represents the radius of the secondary white dwarf and may be given by (Zapolsky and Salpeter 1969; Kieboom and Verbunt 1981)

$$R_2 = 0.0126 \, r \, (M_2/M_\odot)^{-0.283} R_\odot$$

for $0.01 < M_2/M_\odot < 0.3$, (12)

where r (>1) is the ratio of the hot helium core radius to the cold helium core radius (see, e.g., Rappaport et al. 1987). Thus,

we obtain

$$\frac{\dot{M}_2}{M_2} = -\frac{2}{J} \left(\frac{dJ}{dt} \right)_{GR} / [0.283 + H(q)], \quad (13)$$

where the mass transfer rate is given by $-\dot{M}_2$, and the rate of the angular momentum loss due to the gravitational wave radiation is given by

$$\frac{1}{J} \left(\frac{dJ}{dt} \right)_{GR} = -8.28 \times 10^{-10} \left(\frac{M_1}{M_\odot} \right)^3 q(1+q) \left(\frac{a}{R_\odot} \right)^{-4} \text{ yr}^{-1}. \quad (14)$$

The separation of the binary, a , in the equation is obtained by setting $R_2^* = R_2$ in equation (1).

For a sufficiently large value of M_2 , however, equation (13) gives a positive value of \dot{M}_2 . This means that the mass transfer is unstable (Tutukov and Yungelson 1979; Iben and Tutukov 1984; Webbink 1984; Cameron and Iben 1986). Tutukov and Yungelson (1979) (and also Iben and Tutukov 1984; Webbink 1984) further assumed that this dynamical mass transfer disrupts the secondary white dwarf and gets its debris being transformed to a heavy disk rotating around the primary. Recently, Hachisu, Eriguchi, and Nomoto (1986a, b) showed that this is not the case because the energy of the central white dwarf-heavy disk system is higher than that of the double white dwarf system just before the disruption. They, instead, proposed an alternative scenario: a high mass accretion onto the primary first forms a hot envelope around the primary by a shock heating of gas and this hot envelope eventually expands to form a common envelope. This hot

envelope can prevent the following gas from dynamically accreting onto the primary.

After the formation of common envelope, the mass accretion rate (i.e., the mass transfer rate from the secondary to the primary) is constrained to be within the critical mass accretion rate that corresponds to the Eddington luminosity. A further complex situation arises if the primary is a white dwarf: after a small amount of helium ($\sim 0.001 M_{\odot}$) is accreted onto the massive white dwarf primary, a helium shell burning occurs (Kawai, Saio, and Nomoto 1987a) and then the critical mass accretion rate decreases down to

$$\dot{M}_{RG} = 7.2 \times 10^{-6} (M_1/M_{\odot} - 0.6) M_{\odot} \text{ yr}^{-1}$$

for $0.75 < M_1/M_{\odot} < 1.38$, (15)

where \dot{M}_{RG} is the critical mass accretion rate at which the accreting white dwarf becomes red giant-like (Nomoto 1982).

In summary, if equation (13) gives a positive value for \dot{M}_2 or if $|\dot{M}_2|$ is larger than the critical value, which is the Eddington accretion rate for the neutron star primary or \dot{M}_{RG} for the white dwarf primary, the binary system forms a common envelope and the actual mass transfer rate is close to the critical accretion rate.

b) Evolution in the Common Envelope Phase

If the effect of the gravitational wave radiation wins that

of the mass transfer, the common envelope finally fills its outer critical Roche lobe and the dynamical mass outflow occurs from or near the outer Lagrangian point(s). The secondary white dwarf may be disrupted completely. If so, the binary system cannot survive. The effect of the gravitational wave radiation depends strongly on the secondary white dwarf mass, $M_{2,0}$, just at the filling-up of the inner critical Roche lobe. In this subsection, the subscript 0 indicates the beginning of mass transfer.

In order to obtain the upper limit of $M_{2,0}$ for the secondary to survive the common envelope phase, we followed the common envelope evolutions for various sets of the initial masses ($M_{1,0}$, $M_{2,0}$). We used the prescription for the mass transfer rate given in the previous subsection. For neutron star (NS)-white dwarf (WD) binaries, we adopt a value of $6 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ for the Eddington accretion limit onto the NS. The upper limits of $M_{2,0}$ are shown in Figure 3 by solid lines for NS-WD, WD-WD pairs. The possibility of hot helium white dwarf is considered by computing the cases with $\gamma=1.5$ (see eq.[12]) for NS-WD binaries. The dot-dashed line indicates the lower limit of $M_{2,0}$ for the binaries formed by tidal capture (§II). This figure shows that all the NS-WD binaries tidally captured by a neutron star will be disrupted during the common envelope phase. Therefore, such a system cannot be a progenitor of a 4U1820-30 like ultra-compact binary.

The WD-WD binaries with $M_{2,0}$ less than the upper limit value can survive the common envelope phase and become semi-detached binaries with a stable mass transfer driven by the gravitational wave radiation.

IV. EVOLUTION IN THE SEMI-DETACHED PHASE

In the semi-detached phase of the double white dwarf evolutions, the mass transfer rate is smaller than $\dot{M}_{1, \text{RG}}$, for which the helium shell burning in the primary is unstable (Kawai, Saio, and Nomoto 1987b) and shell flashes occur one after another. If the helium shell flash is strong enough, the wind mass loss occurs and a certain amount of envelope mass is lost from the system (Kato, Saio, and Hachisu 1988). According to the numerical results obtained by Kato, Saio, and Hachisu (1988), the ratio of the lost mass to the transferred mass, $\Delta M_\ell / \Delta M_{\text{tr}}$, for $M_1 = 1.3 M_\odot$ may be expressed as

$$\frac{\Delta M_\ell}{\Delta M_{\text{tr}}} = \begin{cases} \exp[0.15 - 1.13 \dot{m} / (10^{-6} M_\odot \text{ yr}^{-1})] - 0.17 & \text{for } \dot{m} < 1.7 \times 10^{-6} M_\odot \text{ yr}^{-1}, \\ 0 & \text{for } \dot{m} \geq 1.7 \times 10^{-6} M_\odot \text{ yr}^{-1}, \end{cases} \quad (16)$$

where $\dot{m} (>0)$ represents the mass transfer rate. The strength of the flashes is probably determined by $\dot{m} / \dot{M}_{1, \text{RG}}$. So we generalize equation (16) for other values of M_1 as

$$\frac{-\dot{M}}{\dot{m}} = \max\{0, \exp[0.15 - 1.13 \frac{\dot{m}}{\dot{M}_{1\text{RG}}} \frac{\dot{M}_{1\text{RG}}(M_1=1.3 M_\odot)}{10^{-6} M_\odot \text{ yr}^{-1}}] - 0.17\}, \quad (17)$$

where $M = M_1 + M_2$ is the total mass of the binary system.

Since the outflowing mass carries away the orbital angular

momentum from the system, the rate of the total angular momentum change is given by

$$\dot{J}/J = \ell \frac{(1+q)^2}{q} \frac{\dot{M}}{M} + \frac{1}{J} \left(\frac{dJ}{dt} \right)_{GR}, \quad (18)$$

where we adopt $\ell = 1.7$ (see §11a). Taking into account the systemic losses of mass and angular momentum, we obtain, instead of equation (13),

$$\frac{\dot{M}_2}{M_2} = \left\{ -\frac{2}{J} \left(\frac{dJ}{dt} \right)_{GR} - \left[2\ell \frac{(1+q)^2}{q} - 3 - H(q) \right] \frac{\dot{M}}{M} \right\} / [0.283 + H(q)]. \quad (19)$$

The actual accretion rate onto the primary is $\dot{M}_1 = \dot{M} - \dot{M}_2$.

Although the mass loss due to the shell flashes occurs intermittently, we consider it as a continuous process in equation (17) for simplicity. Therefore, all values of \dot{M}_1 , \dot{M}_2 , and \dot{m} are considered to be time-averaged values during one cycle of flash, at least, from now on. Then, it is somewhat uncertain what kind of \dot{m} we should use.

Just after the mass loss, the separation of the binary shrinks due to the orbital angular momentum loss and the secondary overflows the inner critical Roche lobe. As a result, the binary system remains a common envelope state for a while. During this common envelope phase, helium is steadily converted into carbon and oxygen (C+O). Since the separation increases due to mass transfer, the binary system finally becomes semi-detached again. During the semi-detached phase, the mass transfer rate is

determined by equation (13). If we adopt \dot{m} which is equal to $-\dot{M}_2$ for $\dot{M}=0$ and is calculated from equation (13), it may give us the lower limit of the mass transfer rate that we can consider. Moreover, this choice of \dot{m} does not include the mass accumulation during the common envelope state just after the mass loss. On the other hand, from the time-averaged treatment point of view, it is reasonable for us to adopt \dot{M}_1 as \dot{m} . The time averaged value of \dot{M}_1 is much larger than \dot{m} which is given by equation (13). Therefore, if we use $\dot{m} = \dot{M}_1$, the mass loss is reduced to some extent and, as a result, we can obtain a much wider parameter region which can produce a 4U1820-30 like system as will be explained below. The actual situation may lie between these two limiting cases. In this paper, therefore, we will discuss the results for both cases.

V. THE FATE OF DOUBLE WHITE DWARFS

The final outcome of mass-transferring double white dwarfs depends on whether the primary is an O+Ne+Mg white dwarf or a C+O white dwarf. We will first discuss the case of O+Ne+Mg white dwarfs and then the case of C+O white dwarfs.

a) ONeMg-He system

The steady helium shell burning converts helium to carbon and oxygen (C+O) atop the O+Ne+Mg core. When the primary mass grows up to $1.38 M_{\odot}$ in a time scale of 10^6 yr, the electron capture on ^{24}Mg triggers the implosion of O+Ne+Mg core and finally forms a neutron star in a quiescent manner (Miyaji et al. 1980; Miyaji and Nomoto 1987). Then, the common envelope phase may appear again, because the critical accretion rate decreases. Therefore, when the primary mass grows up to $1.38 M_{\odot}$, the secondary mass should be less than $\sim 0.1 M_{\odot}$ to survive the second common envelope phase (Fig. 3).

On the other hand, the mass transfer rate should be larger than $\sim 2 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ during the phase when the primary is still a white dwarf. Otherwise, a helium shell detonation occurs rather than a helium shell flash (Kawai, Saio, and Nomoto 1987a). Once the detonation occurs, it may blow up the accumulated envelope mass of helium completely. This means that the primary mass cannot grow up to $1.38 M_{\odot}$.

Using the prescriptions for the mass transfer rate given in §§III and IV, we calculated time evolutions of the binary

starting from various sets of $(M_{1,0}, M_{2,0})$. The initial condition which leads the NS-WD system into surviving the second common envelope phase is below the upper thin solid line in Figure 4. If we use \dot{m} which is calculated from equation (13), \dot{m} becomes smaller than $2 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ for the models below the thin dashed line before the white dwarf primary grows up to $1.38 M_{\odot}$. Then the helium detonation occurs on the white dwarf primary.

For the case of $\dot{m} = \dot{M}_1$, however, the helium detonation never occurs until the secondary mass decreases down to $0.04 M_{\odot}$. (In this paper, we assume that the present mass of the secondary is larger than $0.04 M_{\odot}$.) The lower thin solid line indicates the initial models at which the secondary mass becomes $0.04 M_{\odot}$ just at the neutron star formation. Somewhat below this line, the primary white dwarf will never become a neutron star, and the system will stay a double white dwarf system because the mass of the secondary is insufficient to produce an accretion induced collapse of the primary.

The double white dwarf system with initial parameters in the shaded region can become a system like 4U1820-30 after the primary becomes a neutron star through the accretion induced collapse. Here, the lower mass limit of the secondary (the dash-dot line at the left hand side in Fig. 4) is determined by the lower mass limit for escaping from tidal disruption at the tidal capture (§II), while the upper limit is determined by the condition for surviving the first common envelope (WD-WD) phase

Figure 5 shows an evolution of the binary with $M_{1,0} = 1.260 M_{\odot}$, $M_{2,0} = 0.225 M_{\odot}$, and \dot{m} which is determined by equation (13),

for example. Common envelope phase does not appear in this case. The epoch of the neutron star formation is indicated by an arrow. The systemic mass loss due to the helium shell flashes occurs from $\log t(\text{yr}) = 4.17$ to the neutron star formation. The mass transfer rate \dot{m} calculated by equation (13) is much smaller than \dot{M}_1 and this difference mainly makes the difference in the possible parameter region of Figure 4. If we use $\dot{m} = \dot{M}_1$, we obtain different time evolutionary tracks, but the neutron star formation and the resultant evolution are essentially the same. The mass of the secondary is indicated on the upper horizontal axis.

b) CO-He system

If the accretion rate on a C+O white dwarf is sufficiently high, an off-center carbon ignition occurs (Nomoto and Iben 1985; Kawai, Saio, and Nomoto 1987a) and the whole C+O white dwarf is converted into an O+Ne+Mg white dwarf (Saio and Nomoto 1985, 1987). After the conversion, the condition for the binary system to become a 4U1820-30 like binary is essentially the same as that discussed in the preceding subsection. The boundary for the occurrence of the off-center carbon ignition is given by the dotted line in Figure 6. The shaded region indicates the possible parameter space in which a 4U1820-30 like binary system can be born.

If the off-center carbon flash does not occur, on the other hand, its final outcome is a supernova (SN) when $M_{1,0} + M_{2,0} \gtrsim 1.46\text{--}1.48 M_\odot$ (depends on the choice of \dot{m}) by inducing a carbon

deflagration at the center of C+O white dwarf as shown in Figure 6. Whether this supernova explosion becomes a Type I supernova or it leaves a neutron star depends on the thermal history of the progenitor white dwarf (see, e.g., Nomoto 1987). If it can leave a neutron star, the binary system might become a system like 4U1820-30. Then the possible parameter region is the same as for the O+Ne+Mg white dwarf-helium white dwarf pairs in Figure 4. When $M_{1,0} + M_{2,0} \lesssim 1.48 M_{\odot}$, a double detonation supernova occurs for the relatively low accretion rates, i.e., $\dot{m} < 2 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ (Fujimoto and Sugimoto 1982; also see Nomoto, Thielemann, and Yokoi 1984; Kawai, Saio, and Nomoto 1987a). Otherwise, two white dwarfs can remain a double white dwarf system.

VI. DISCUSSION

a) The origin of massive O+Ne+Mg white dwarfs

In the previous section, we found that a massive ($>1.2 M_{\odot}$) white dwarf is necessary to produce a 4U1820-30 like ultra-compact binary. However, from the binary evolutionary point of view, O+Ne+Mg white dwarfs or C+O white dwarfs having masses larger than $1.2 M_{\odot}$ are very unlikely (Iben and Tutukov 1984; Webbink 1984). This range may give a very small possibility, although we cannot say anything about the statistics since we have found only one source like 4U1820-30.

Therefore, we propose another way to produce such a massive O+Ne+Mg white dwarf. Double C+O white dwarfs, which are the most likely products of intermediate-mass binary evolutions (Iben and Tutukov 1984; Webbink 1984), merge into one body getting its debris being transformed into a spread disk rotating around the merged white dwarf (Hachisu, Eriguchi, and Nomoto 1986a). It can be expected that the merged white dwarf becomes an O+Ne+Mg white dwarf, because the dynamically rapid mass accretion during the merging can induce an off-center carbon burning by a compressional heating on the surface of the more massive white dwarf (Saio and Nomoto 1985, 1987).

b) Mass Loss at the Ignition of Carbon Shell Burning

During the common envelope phase, helium accreted on the primary is converted to carbon and oxygen by the steady helium

shell burning. Thus the C+O matter accumulate onto the primary at the same rate as the helium accretion. Since the accretion rate onto the primary is about $5 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ ($=\dot{M}_{1,\text{RG}}$ in eq.[15]) for the case of $M_1 \sim 1.3 M_{\odot}$ in the common envelope phase, an off-center carbon ignition occurs in the white dwarf primary when M_1 reaches $1.3 M_{\odot}$ if it is a C+O white dwarf or when the C+O envelope becomes thicker than $\sim 0.02 M_{\odot}$ if it is an O+Ne+Mg white dwarf (Kawai, Saio, and Nomoto 1987b). According to the calculation by Saio and Nomoto (1985, 1987) for the accretion rate of $1 \times 10^{-5} M_{\odot} \text{ yr}^{-1}$, the envelope of the primary expands over the outer critical Roche lobe and about 20% of the mass above the ignition point is ejected from the binary.

The outflowing mass carries away the orbital angular momentum. Since the expansion velocity of the primary envelope is much slower than the orbital velocity of the binary, the mass is lost from or near the outer Lagrangian point L_2 and the orbital angular momentum is reduced down to

$$J_f = J_i \exp[\ell \Delta M_1 / M_2] [(M_1 + \Delta M_1) / M_1]^{\ell}, \quad (20)$$

where J_f and J_i are the final and the initial orbital angular momentum, respectively, and ΔM_1 (< 0 , negative) is the mass of the envelope matter lost from the primary. Since about 20% decrease in the separation leads the secondary into filling the outer critical Roche lobe, this critical value of mass loss for surviving can be estimated as $\Delta M_{1,\text{cr}} \sim -(0.11/\ell) M_2$. Note that if ΔM_1 is small, the decrease in the separation mainly stems from

the reduction of the total angular momentum (see eq.[5]). If we adopt $M_2 \simeq 0.2 M_\odot$ and $\ell=1.7$, we obtain $\Delta M_{1,cr} \sim -0.012 M_\odot$. Therefore, if more than half of the mass above the ignition shell is lost, the secondary overfills the outer critical Roche lobe and will be disrupted in a dynamical time scale.

As mentioned above, only 20% of the envelope mass is lost for $\dot{M}_1 = 10^{-5} M_\odot \text{ yr}^{-1}$ and the secondary will not be disrupted in this case. Although we do not know how much mass is lost for $\dot{M}_1 \sim 5 \times 10^{-6} M_\odot \text{ yr}^{-1}$, the binary may be disrupted for this case because the off-center ignition is more violent for smaller accretion rate. If it is the case, the possible parameter region of the progenitor of 4U1820-30 like binaries disappears for the C+O primaries. However, we shall note here that for most part of the shaded region for O+Ne+Mg primaries in Figure 4, the carbon ignition never occurs.

c) Mass Loss at the Neutron Star Formation

Some amount of mass may be ejected by a rebounding shock at the neutron star core formation. It can be expected that the velocity of ejected matter is much faster than the orbital velocity ($a\Omega \sim 1400 \text{ km s}^{-1}$). Then, the angular momentum loss per unit mass is estimated to be $\ell a^2 \Omega$ and $\ell \sim q^2 / (1+q)^2$. When we use the values of $M_1 = 1.38 M_\odot$ and $M_2 = 0.1 M_\odot$, the total angular momentum loss from the system is very small ($\sim 2\%$) even if $0.4 M_\odot$ is ejected from the primary. As a result, the separation increases (up to 40% for $\Delta M_1 = -0.4 M_\odot$) and the secondary can

reside inside the inner critical Roche lobe. This effect cannot, therefore, disrupt the secondary. The secondary can fill again its inner critical Roche lobe after 1×10^6 yr due to the orbital angular momentum loss by the gravitational wave radiation for this case of $\Delta M_1 = -0.4 M_\odot$.

d) Age of the Neutron Star

The strength of the magnetic field gives us an information of the age of the neutron star. Ebisuzaki (1987) estimated an upper limit of the magnetic field to be 3×10^{10} gauss from the rising time of the X-ray bursts. The decay time of magnetic fields of neutron stars is roughly estimated to be $10^6 - 10^7$ yr (Flowers and Ruderman 1977; Taam and van den Heuvel 1986). If the neutron star had a strong magnetic field of $\sim 10^{12}$ gauss at its birth time, this upper limit value is marginal for our scenario because the age of the neutron star must not be older than 10^7 yr for 4U1820-30 system. However, its initial magnetic field may be weak as expected in the millisecond pulsars (see, e.g., the reviews by Backer 1984 and van den Heuvel 1984). If it took a long time before the O+Ne+Mg white dwarf captured a red giant, the magnetic field of the white dwarf may decay to much extent. Then the resultant magnetic field of the neutron star might also be weak. At the present time, there is no information that the neutron star of 4U1820-30 is older than 10^7 yr.

VII. CONCLUSION

Some intermediate-mass binaries are evolving to form double C+O white dwarfs after two stages of mass-exchange. These two C+O white dwarfs are approaching each other due to the loss of the orbital angular momentum by the gravitational wave radiation and finally merge into one body after the less massive component fills its inner critical Roche lobe. The final outcome of merging is an O+Ne+Mg white dwarf or a C+O white dwarf and a spread disk rotating around the merged white dwarf. If the white dwarfs thus formed capture a red giant in the core of the globular cluster, a new pair of O+Ne+Mg white dwarf and helium white dwarf or of C+O white dwarf and helium white dwarf is formed after the hydrogen-rich envelope of the captured star is stripped off. We have followed the evolution of such pairs of double white dwarfs.

We have found that the pairs of C+O and helium white dwarfs may not produce the present pair of neutron star and small mass white dwarf like 4U1820-30. Rather it might produce a supernova. For the pairs of O+Ne+Mg and helium white dwarfs, however, they may result in a pair of neutron star and small mass white dwarf by inducing a collapse of white dwarf.

In the globular cluster NGC 6624, the turn-off mass is $0.8 M_{\odot}$ and the resultant core mass after stripping-off of the envelope mass is $0.2-0.4 M_{\odot}$. We can expect that when the mass of helium white dwarf is between $0.18-0.27 M_{\odot}$ and the O+Ne+Mg white dwarf has mass between $1.2-1.3 M_{\odot}$, these system can become a system like 4U1820-30 through the accretion induced collapse of the massive white dwarf.

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FIGURE CAPTIONS

Fig. 1.—The illustration of three types of captures. Case I: the captured star overflows the outer critical Roche lobe and the unstable mass outflow ensues. At the same time, the accretion onto the primary can form a hot envelope around it (shaded). Case II: the radius of the captured star is between the outer and the inner critical Roche lobe and the mass transfer from the secondary to the primary occurs. The rapid mass accretion onto the primary also forms a hot envelope around it (shaded). Case III: the captured star can reside inside the inner critical Roche lobe and no resultant mass flow takes place before it evolves toward a red giant.

Fig. 2.—The maximum values of the effective radii of the outer and the inner critical Roche lobes just after the envelope mass is entirely stripped off (case I) are plotted against the helium core mass. If the radius of the hot helium core is larger than the outer critical Roche lobe, the core mass overflows the outer critical Roche lobe further and will be tidally disrupted. The lower limit of surviving core mass is $0.176 M_{\odot}$ if $M_{1,0} = 1.4 M_{\odot}$ and $0.198 M_{\odot}$ if $M_{1,0} = 1.0 M_{\odot}$. The maximum shrinking time to semi-detached phase by the gravitational wave radiation is also plotted. When $M_2(\text{He}) \lesssim 0.31 M_{\odot}$ (for $M_1 = 1.4 M_{\odot}$), the binary system can always become semi-detached again in the cosmic age. Here, the initial secondary mass is fixed to be $0.8 M_{\odot}$.

Fig. 3.—The upper limits of the initial secondary mass $M_{2,0}$ to avoid disruption during the common envelope phase are shown for NS-WD and WD-WD pairs (solid lines). The parameter γ is the ratio of the hot core radius to the zero-temperature helium white dwarf radius with the same mass. The dash-dot line indicates the lower limit of $M_{2,0}$ for the binaries by tidal capture (§II). This figure shows that all the NS-WD pairs formed by tidal capture are eventually disrupted.

Fig. 4.—The possible parameter range of O+Ne+Mg white dwarf-helium white dwarf system which can produce a pair of neutron star and small mass white dwarf is shown in the $M_{1,0}$ - $M_{2,0}$ plane, where $M_{1,0}$ and $M_{2,0}$ are the initial masses of the O+Ne+Mg white dwarf and the helium white dwarf, respectively. The upper mass limit of O+Ne+Mg white dwarfs is $1.38 M_{\odot}$. The thick solid line is the same as the solid WD-WD line with $\gamma=1$ in Fig. 3. The dash-dot line has the same meaning as in Fig. 3. Models on the right side to the dotted line form a common envelope after a helium shell burning occurs. Models in shaded region can theoretically be a progenitor of 4U1820-30.

Fig. 5.—Evolutionary changes of the three mass transfer rates, $-\dot{M}_2$, \dot{M}_1 , and \dot{m} , and the orbital period, P_{orb} , for $(M_{1,0}, M_{2,0}) = (1.260 M_{\odot}, 0.225 M_{\odot})$. The time, t , is measured from the beginning of the semi-detached phase. The upper horizontal axis indicates the change of the secondary mass, M_2 .

Fig. 8.—The final outcome of C+O white dwarf and helium white dwarf system. If $\dot{M}_2 < 2 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$, an off-center carbon burning never occurs. When the C+O white dwarf mass reaches $1.4 M_{\odot}$, carbon deflagration can ignite at the center and the deflagration wave propagates outward. This results in the explosion of the white dwarf (supernova; SN). If an off-center carbon flash occurs (models on the right side to the dotted line), on the other hand, the C+O white dwarf can become an O+Ne+Mg white dwarf. Only a very small region (shaded) may be expected to produce the present pair of neutron star and white dwarf.

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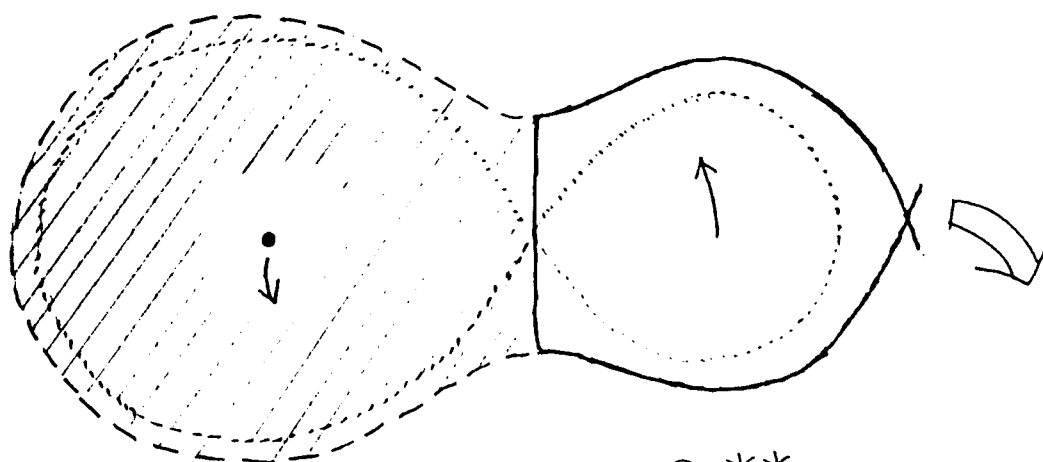
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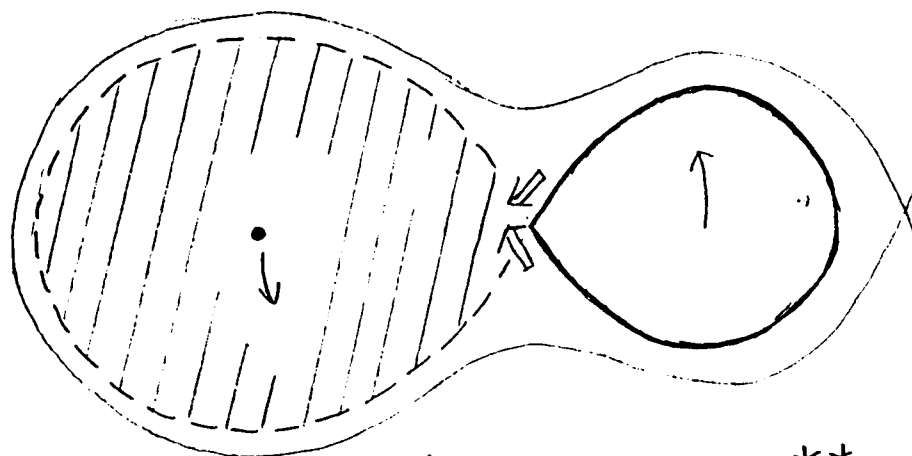
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(I)



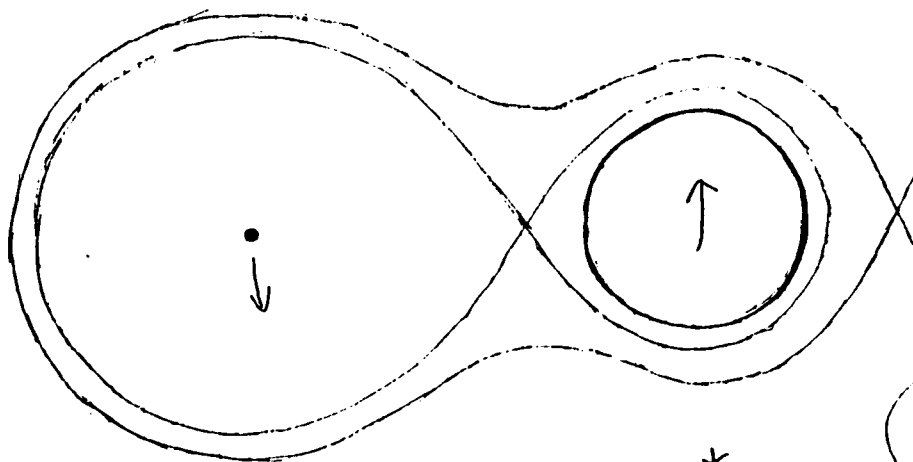
$$R_2 > R_2^{**}$$

(II)



$$R_2^* < R_2 < R_2^{**}$$

(III)



$$R_2 < R_2^*$$

(Fig. 1)

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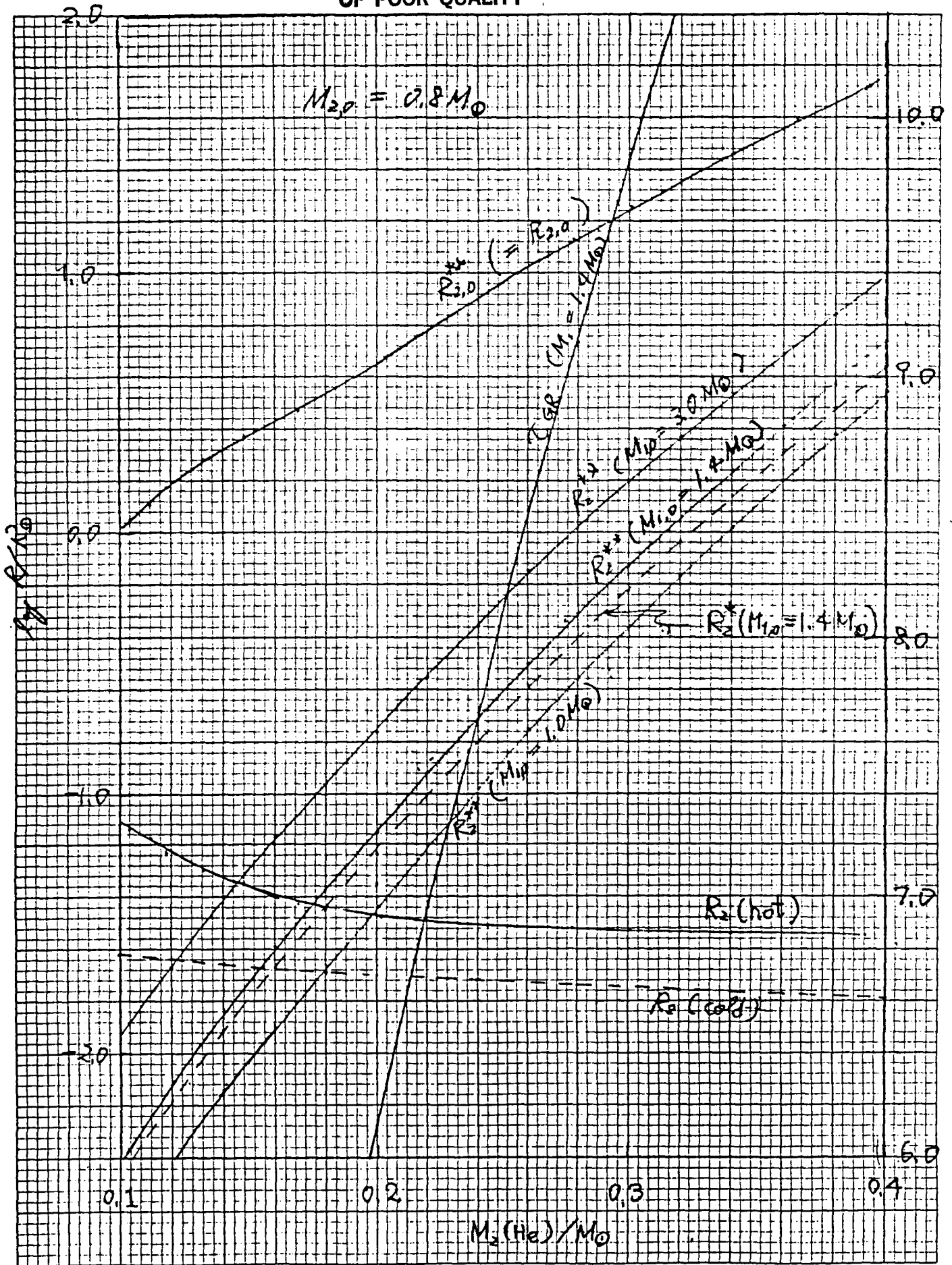
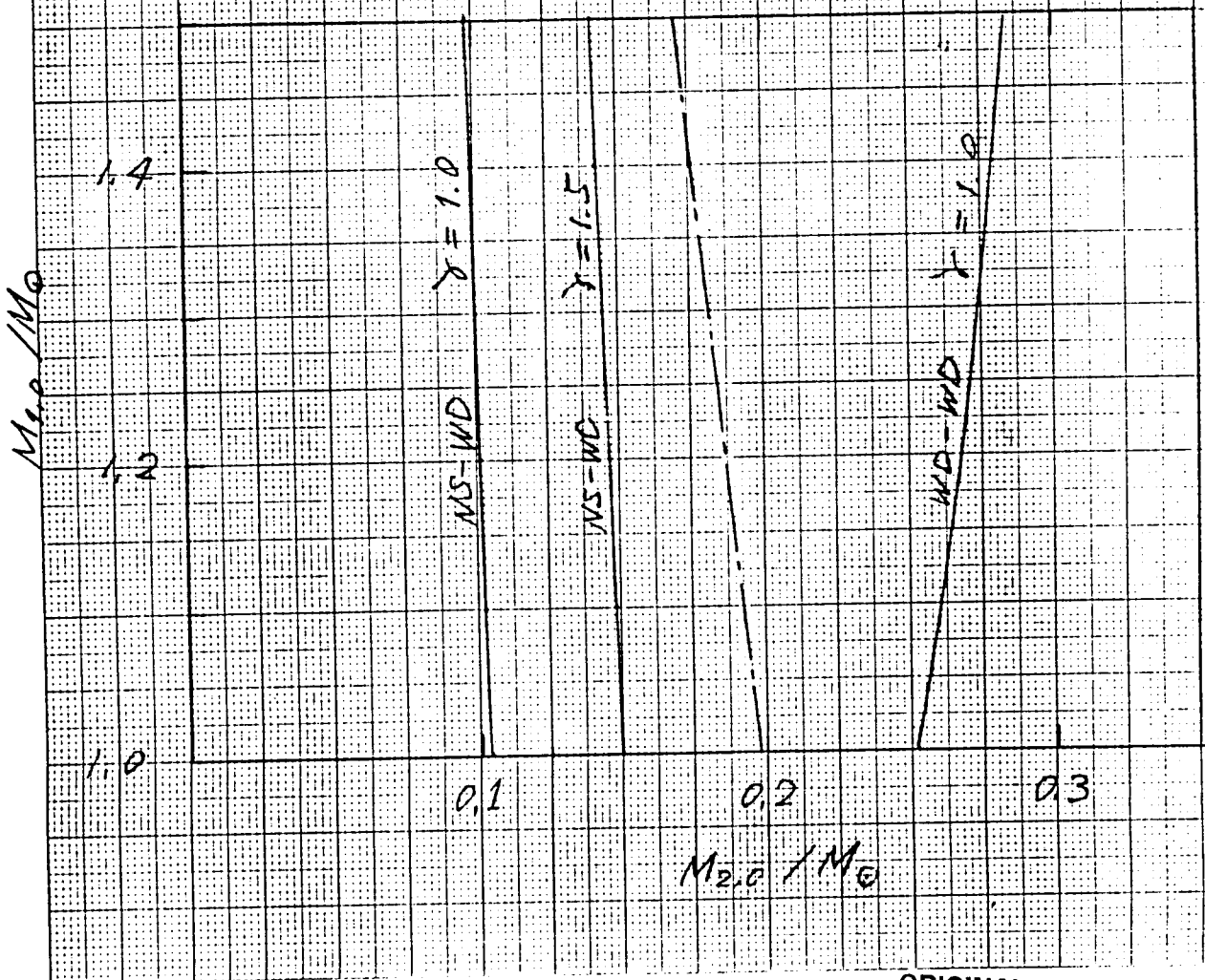


Fig. 2.



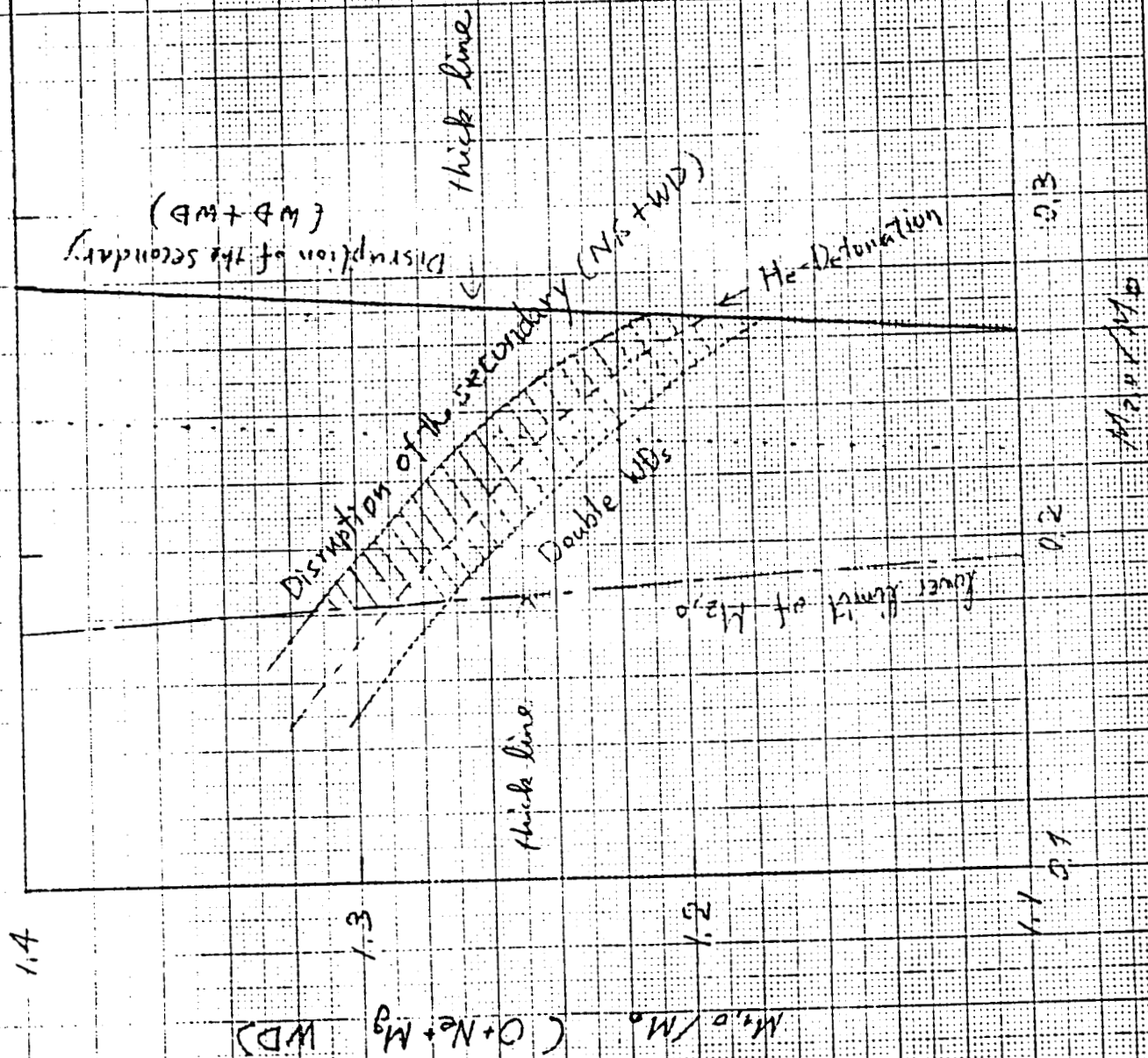
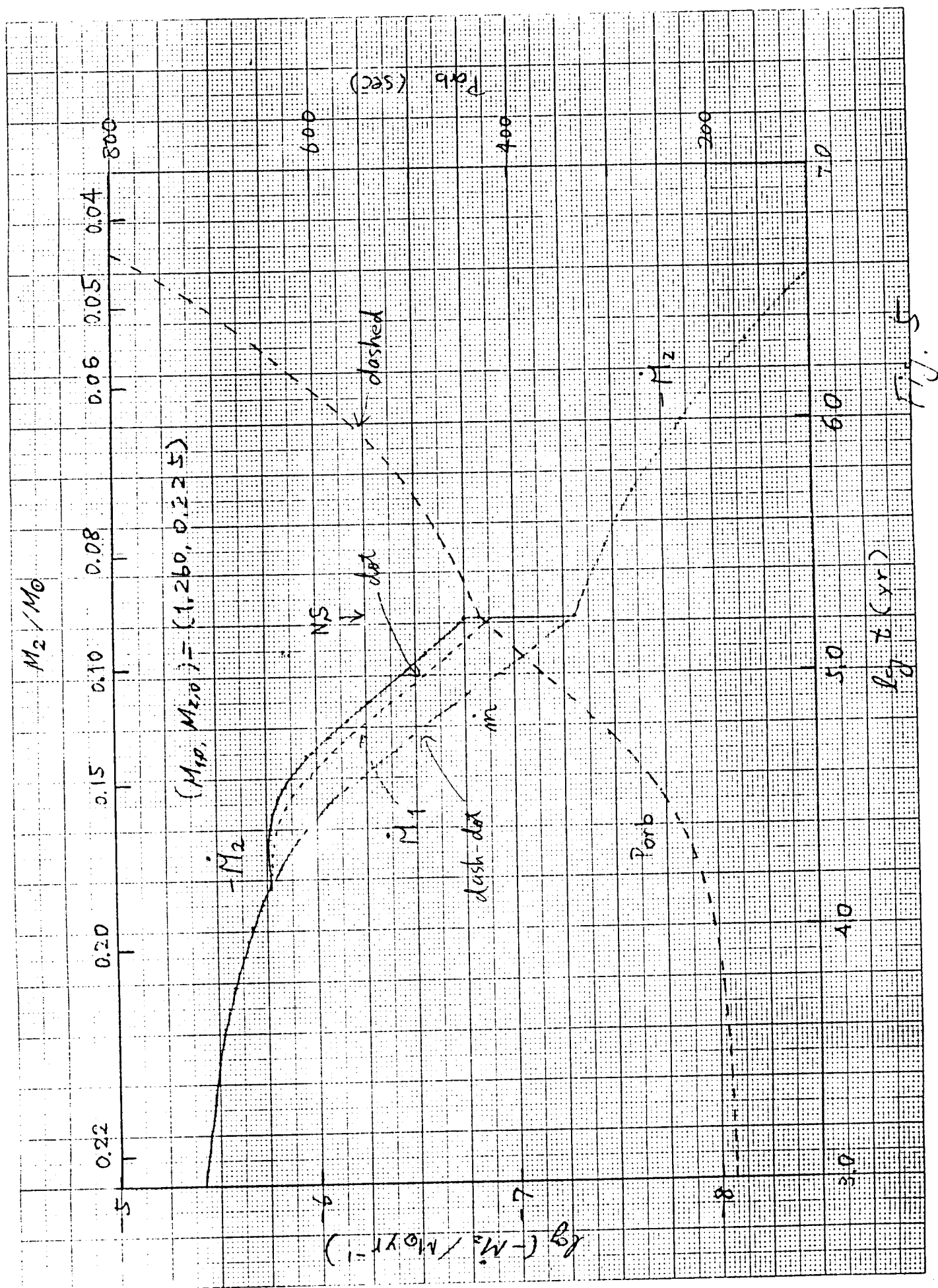


Fig. 1



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